Abstracts

Colin de Verdière, **Yves** - *Spectral asymptotics for sub-Riemannian Laplacians*

I will describe some results obtained with Emmanuel Trélat and Luc Hillairet. I will resctrict to the case of dimension 3 and I will discuss the contact case and the generic case (Martinet singularities).

Guillarmou, **Colin** - *Relations between classical resonances and quantum resonances in constant negative curvature*

In this minicourse, we will define the notion of Ruelle resonance for the geodesic flow on a constant negatively curved manifold M. These are essentially eigenvalues of the vector field X generating the geodesic flow on the unit tangent bundle SM on certain functional spaces. We will show an explicit relation between these eigenvalues of X and the corresponding eigenfunctions with eigenvalues of some quantum operators, typically some Laplacians on certains bundles on M. We will also explain how quantum ergodicity is related to equidistributions of traces of spectral projectors (or Patterson-Sullivan distributions) of Ruelle resonances at high frequencies.

Le Masson, **Étienne** - *Quantum Chaos in the Benjamini-Schramm limit*

One of the fundamental problems in quantum chaos is to understand how high-frequency waves behave in chaotic environments. A famous but vague conjecture of Michael Berry predicts that they should look on small scales like Gaussian random fields. We will first give an overview of this question and introduce the notion of Benjamini-Schramm convergence of manifolds, explaining how it can give a precise formulation of Berry's conjecture. The Benjamini-Schramm convergence includes the high-frequency limit as a special case but provides a much more general framework. Based on this generalised formulation, we will expand the scope and consider a case where the frequencies stay bounded and the size of the manifold increases instead. In this alternative setting we will explain the proof of a quantum ergodicity theorem and how it relates to the random wave conjecture.

Lefeuvre, **Thibault** - *The marked length spectrum of negatively-curved manifolds*

The marked length spectrum of a negatively-curved manifold is the collection of lengths of closed geodesics, differentiated by their free homotopy classes. It was conjectured by Burns and Katok in the ‘80s that it should parametrize the set of isometry classes of the manifold — which is not the case of the length spectrum (the collection of lengths, regardless of the homotopy), as proved by Vigneras. In the ‘90s, Croke and Otal proved this conjecture in dimension two but there has not been much progress since and the question remains open. I will present a proof of a local version of the conjecture which holds in any dimension and generalizes to Anosov geodesic flows under some assumptions. This is a joint work with Colin Guillarmou.

Rivière, **Gabriel** - *Quantum ergodicity: old and new results*

The quantum ergodicity theorem is a classical result on the distribution of Laplace eigenfunctions on negatively curved manifolds. In a first lecture, I will describe the context in which this theorem appears, its statement and some conjectures related to it. In a second lecture, I will discuss some background materials related to microlocal analysis. In a third lecture, I will build on the tools of the second lecture and give a proof of the quantum ergodicity theorem. In a fourth lecture, I will discuss some recent advances due to Anantharaman, Bourgain, Dyatlov, Lindenstrauss, Hassell, Nonnenmacher and Jin.

Sabri, **Mostafa** - *Norm estimates for eigenfunctions of general graphs*

Recent years have seen much interest in the study of delocalization on graphs. One may show the eigenfunctions in a spectral region are delocalized using different criteria. For instance, one can try to show the eigenfunctions have large support. One can try to prove quantum ergodicity, showing that most eigenfunctions equidistribute on the graph in some sense. Yet another criterion is to give upper bounds on the supremum norms of the eigenfunctions, and more generally the p-norms for $p > 2$.

In this talk, I will discuss some results on the norms of eigenfunctions of a Schrödinger operator H on a large finite graph G. It is assumed that (G, H) is somehow of limited complexity - this class includes N-lifts of some base graph. The estimates are valid for all eigenfunctions of the graph, except for a finite set of exceptional energies.

Joint work with Étienne Le Masson.